$2^{\frac{1}{\log_{n!} 2}} = 2^{\log_2 n!}$ 2^x $\downarrow$ $n!$ $x = \log_2 n!$

$2^x = n!$

$36^{\log_6 2n}$ $\log_6 2n = x$

36^x $6^x = 2n$

6^{2x} $6^{2x} = 4n^2$

Logorithms

iterated Logorithm

k times

$\log(\log(\log \ldots (x))))$

$\lg^* n = \min\{i \geq 0 \mid \lg^{(i)} n \leq 1\}$

for any value n, it takes i itterations to reduce to the vale 1

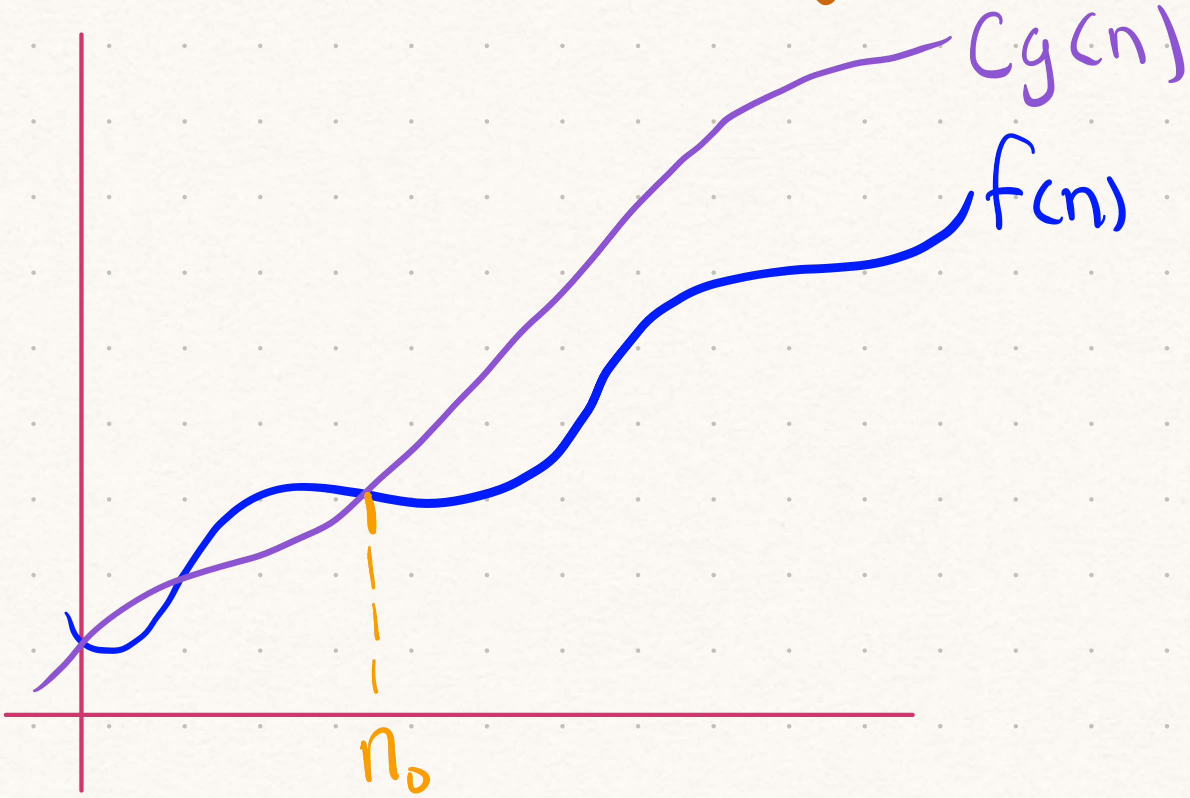
$\log^* 2 = 1$ $\log^* 16 = 3$

$\log^* 4 = 2$ $\log^* (2^{16}) = 4$

Big O Notation Review

Upper bound

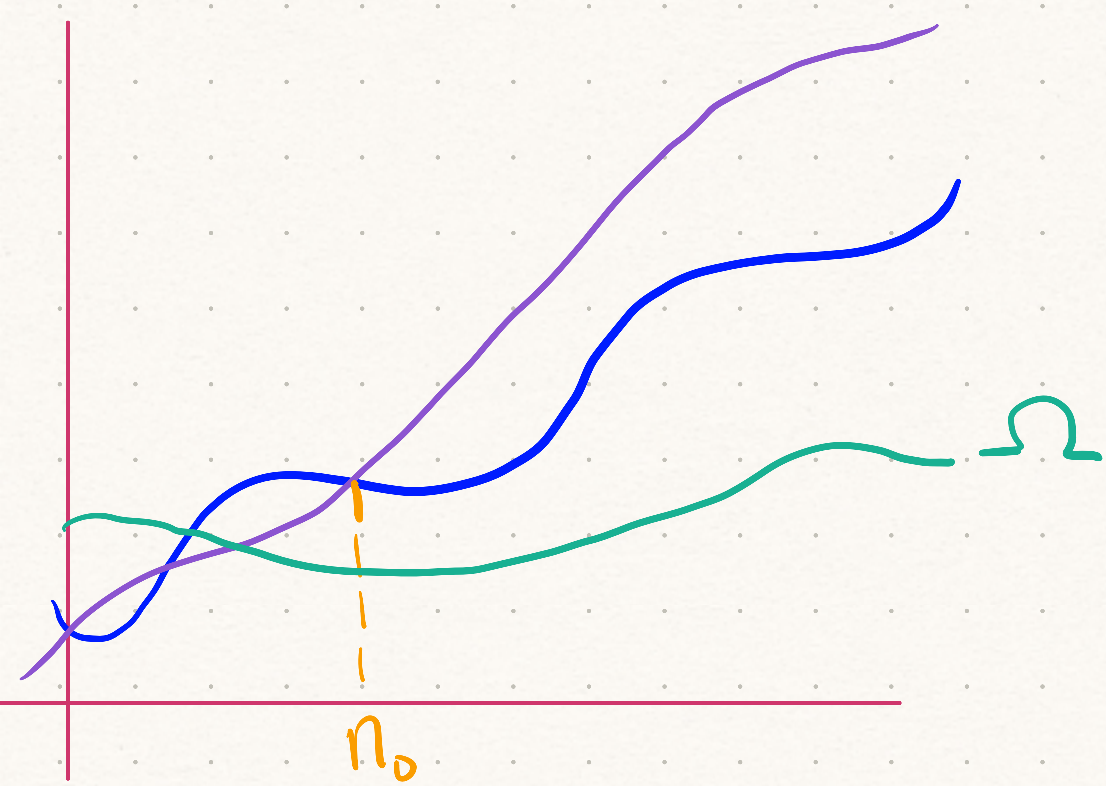
$$O(g(n)) = \{ f(n) \mid \exists c, n_0 \text{ s.t. } 0 \le f(n) \le c\,g(n) \; \forall n \ge n_0 \}$$



Big Omega Notation

lower Bound

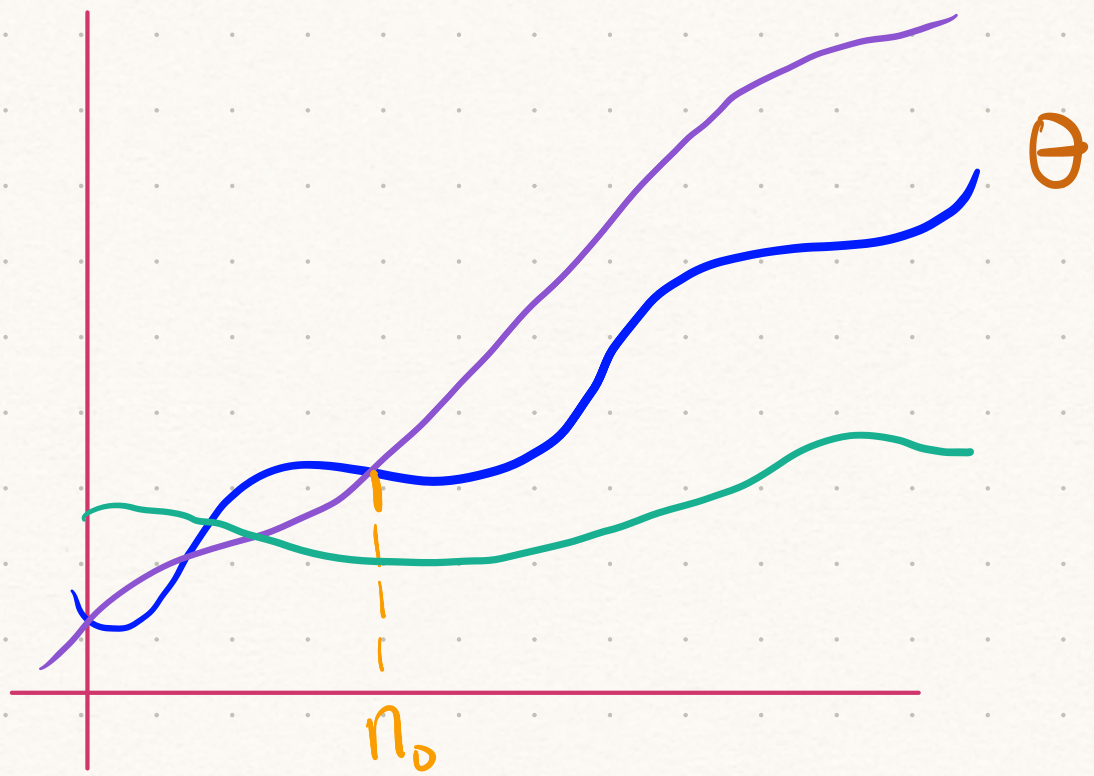
$$\Omega(g(n)) = \{f(n) \mid \exists c, n_0 \text{ s.t. } 0 \le c\,g(n) \le f(n)\ \forall n \ge n_0\}$$



Big Theta Notation

assymptotic bound

$$\Theta(g(n)) = \{ f(n) \mid \exists\, c_1, c_2, n_0 \text{ st. } 0 \le c_1 g(n) \le f(n) \le c_2 (g(n)) \ \forall n \ge n_0 \}$$



Theorem: $f(n) = \Theta(g(n))$ iff
$f(n) = O(g(n))$ & $f(n) = \Omega(g(n))$

Suppose $f(n) = \Theta(g(n))$, that is
$\exists\, c_1, c_2, n_0$ st. $0 \leq c_1 g(n) \leq f(n) \leq c_2 (g(n))$ $\forall n \geq n_0$, thus
Since $0 \leq c_1 (g(n)) \leq f(n)$ which means $\Omega(g(n))$ of $f(n)$ & sine
$0 \leq f(n) \leq c_2 g(n)$ then $O(g(n))$ of $f(n)$, thus proving $\Theta(g(n))$

Suppose $f(n) = O(g(n))$ & $f(n) = \Omega(g(n))$,
that is $\exists\, c_1, c_2, n_0$, such that
Since $f(n) = O(g(n))$, $0 \leq f(n) \leq c_2 g(n)$,
and since $0 \leq c_1 g(n) \leq f(n)$, thus
$0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$, thus
supplying the definition of $\Theta(g(n))$
& thus proving $f(n) = \Theta(g(n))$ iff
$f(n) = O(g(n))$ & $f(n) = \Omega(g(n))$.